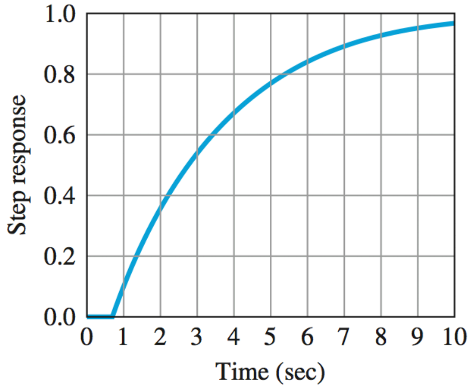
Control Systems : Set 3 : PID (2) - Solutions

Prob 1 | The unit-step response of a paper machine is shown in the figure below, where the input to the system is stock flow onto the wire and the output is the basis weight of the paper (i.e., the thickness). Find the proportional, PI, and PID-controller parameters using the Ziegler-Nichols method.



From the step response, we can measure the delay to be around $L = 0.6$s, and eyeball the maximum slope to be around $a = 0.3$.
From the lecture notes we get:

$$
\begin{array}{llll}
P & K_P = \dfrac{1}{aL} = 5.6 & & \\
PI & K_P = \dfrac{0.9}{aL} = 5 & T_i = 3.3L = 2 & \\
PID & K_P = \dfrac{1.2}{aL} = 6.7 & T_i = 2L = 1.2 & T_d = 0.5L = 0.3
\end{array}
$$

Prob 2 | A system has the transfer function

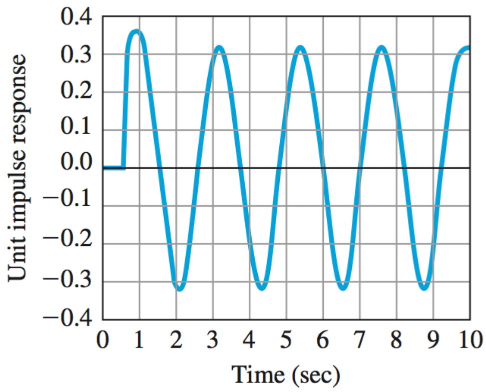
$$G(s) = \frac{e^{-2s}}{3s+1}$$

Find the PID-controller parameters using the Ziegler-Nichols tuning rules.
Hint: The maximum slope of a first-order system is given by the inverse of the time constant.

The time constant here is 3, which tells us that the maximum slope is $a = 0.33$. We can see from the transfer function that the system delay is $L = 2$ seconds.
From the notes we have:

$$
\begin{array}{llll}
PID & K_P = \dfrac{1.2}{aL} = 1.8 & T_i = 2L = 4 & T_d = 0.5L = 1
\end{array}
$$

Prob 3 | Using proportional feedback control, control designers have obtained a closed-loop system with the unit impulse response shown in the figure below. When the gain is $K_{pc} = 8.556$, the system is on the verge of instability. Determine the proportional-, PI-, and PID-controller parameters according to the Ziegler-Nichols method.



We estimate the period of the response from the figure as $T_c = 2.3$s.
From the lecture notes we get:

P	$K_P = 0.5K_{pc} = 4.3$		
PI	$K_P = 0.45K_{pc} = 3.9$	$T_i = 0.83T_c = 1.9$	
PID	$K_P = 0.6K_{pc} = 5.1$	$T_i = 0.5T_c = 1.2$	$T_d = 0.125T_c = 0.3$